

# Magnetotransport along a boundary: From coherent electron focusing to edge channel transport

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We study theoretically how electrons, coherently injected at one point on the boundary of a two-dimensional electron system, are focused by a perpendicular magnetic field  $B$  onto another point on the boundary. Using the non-equilibrium Green's function approach, we calculate the generalized 4-point Hall resistance  $R_{xy}$  as a function of  $B$ . In weak fields,  $R_{xy}$  shows the characteristic equidistant peaks observed in the experiment and explained by classical cyclotron motion along the boundary. In strong fields,  $R_{xy}$  shows a single extended plateau reflecting the quantum Hall effect. In intermediate fields, we find superimposed upon the lower Hall plateaus anomalous oscillations, which are neither periodic in  $1/B$  (quantum Hall effect) nor in  $B$  (classical cyclotron motion). The oscillations are explained by the interference between the occupied edge channels, which causes beatings in  $R_{xy}$ . In the case of two occupied edge channels, these beatings constitute a new commensurability between the magnetic flux enclosed within the edge channels and the flux quantum. Introducing decoherence and a partially specular boundary shows that this new effect is quite robust.

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## I. INTRODUCTION

When electrons are injected coherently at one point on the boundary of a two-dimensional electron gas (2DEG), they can be focused by a perpendicular magnetic field  $B$  onto another point of that boundary.<sup>1</sup> In the classical regime, resonances are observed when a multiple of the cyclotron diameter equals the distance between the injecting and collecting point contacts. For large Fermi wavelength and long phase coherence length additional interference effects are observed. This regime of *coherent electron focusing* has been studied for the first time by van Houten et al.<sup>2</sup> Recently, the effects of disorder<sup>3</sup> and spin-orbit interaction<sup>4–8</sup> were studied. It was also suggested to investigate the structure of graphene edges by coherent electron focusing.<sup>9</sup> Moreover, a 2DEG in a strong magnetic field shows the quantum Hall effect, which is explained by the transport through *edge channels* straight along the boundary of the system.<sup>10</sup>

Here, we study theoretically the properties of the focusing experiment emerging at the transition from the classical cyclotron motion to the quantum Hall edge channel transport. Using the non-equilibrium Green's function (NEGF) approach,<sup>10–12</sup> we calculate the generalized 4-point Hall resistance  $R_{xy}$  as a function of a perpendicular magnetic field  $B$ . In weak magnetic fields, the focusing spectrum shows equidistant peaks (see Fig. 1), which can be explained by classical cyclotron orbits. In strong magnetic fields, the typical fingerprint of the quantum Hall effect can be observed, i.e. an extended Hall plateau with  $R_{xy} = h/2e^2$ . Additionally and somewhat unexpectedly, in intermediate fields, instead of lower Hall plateaus we find oscillations, which are neither periodic in  $1/B$  (quantum Hall effect) nor periodic in  $B$  (classical cyclotron motion). These oscillations can be explained by the interference of the occupied edge channels causing beatings in  $R_{xy}$ .

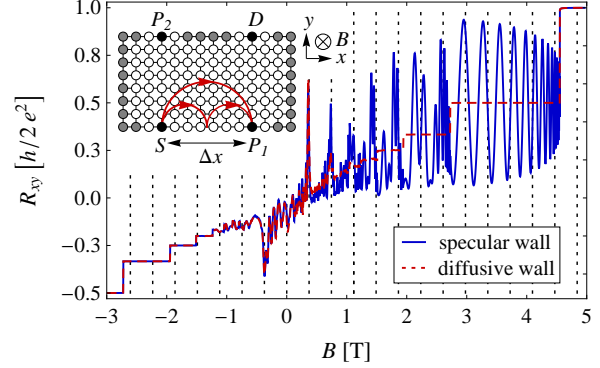


FIG. 1: (Color online) The Hall resistance  $R_{xy} = \frac{\mu_{P1} - \mu_{P2}}{eI_{SD}}$  as a function of a magnetic field  $B$  for the system sketched in the inset. In weak fields,  $R_{xy}$  shows equidistant focusing peaks indicated by dashed vertical lines, when a multiple of the cyclotron diameter equals  $\Delta x$ . In intermediate fields, anomalous oscillations appear, which are neither periodic in  $1/B$  nor in  $B$ . A single Hall plateau is found in large fields, whereas lower Hall plateaus can only be seen when specular reflections are suppressed by an absorbing diffusive wall between  $S$  and  $P_1$ .

## II. SYSTEM

We consider a GaAs-AlGaAs heterojunction, where a 2DEG is formed at the interface of the two semiconductors. The 2DEG is described by the Hamiltonian

$$H_{2DEG} = \frac{(\mathbf{p} - e\mathbf{A})^2}{2m}, \quad (1)$$

where  $m = 0.07m_e$  is the effective mass of the electrons, and  $\mathbf{A} = By\mathbf{e}_x$  is the vector potential of a homogeneous magnetic field  $\mathbf{B} = -B\mathbf{e}_z$ , which is perpendicular to the 2DEG, see the inset of Fig. 1. The Hamiltonian is

approximated by finite differences<sup>13</sup>

$$H_{2\text{DEG}}^{\text{FDA}} = \sum_{\mathbf{r}_i, \mathbf{r}_j} t e^{i\pi \frac{eB}{h} (x_j - x_i)(y_j + y_i)} |\mathbf{r}_i\rangle \langle \mathbf{r}_j|, \quad (2)$$

where the sum is over nearest neighbors in a square lattice of sites at a distance  $a = 5 \text{ nm}$ , and  $t = \frac{\hbar^2}{2ma^2} \approx 21.8 \text{ meV}$ . This approximation is valid when the magnetic flux through a unit cell  $Ba^2$  is much smaller than a flux quantum  $h/e$ . We assume that experimentally, the influence of the temperature is negligible and thus, we set the temperature to  $T = 0 \text{ K}$ . For simplicity, we also assume that the spin splitting is not resolved.<sup>24</sup>

The system size is  $800 \text{ nm} \times 500 \text{ nm}$ . Metallic contact regions are attached at the boundaries of the system, separated by a distance  $\Delta x = 500 \text{ nm}$ , see the inset of Fig. 1. The chemical potential is set to  $\mu = 0.5t \approx 10.9 \text{ meV}$ , corresponding to the carrier density  $n_{2\text{D}} \approx 3.3 \cdot 10^{11} \text{ cm}^{-2}$ . The chemical potential of the voltage probes  $\mu_{P_i}$  is calculated by assuming an infinitesimal bias voltage between  $S$  and  $D$ , and by using the constraint that voltage measurements are done without a current flow through the voltage probes. The voltage between  $P_1$  and  $P_2$  divided by the current between  $S$  and  $D$  gives the generalized 4-point Hall resistance<sup>2</sup>  $R_{xy} = \frac{\mu_{P_1} - \mu_{P_2}}{eI_{SD}}$ . Details of the NEGF calculation of the current and the chemical potential can be found in the following section.

### A. Details of the calculations

Following the NEGF approach, as described in detail by Datta,<sup>10–12</sup> the transmission from the  $j$ th to the  $i$ th contact is given by

$$T_{ij} = 4\text{Tr} \left( \text{Im}(\Gamma_i) G \text{Im}(\Gamma_j) G^+ \right), \quad (3)$$

where the Green's function is defined as

$$G = \left[ E - H - \sum_{k=1}^{N_c} \Gamma_k \right]^{-1}. \quad (4)$$

The influence of each of the  $N_c$  contacts is taken into account by an imaginary self-energy

$$\Gamma_k = -i\eta \sum_{\mathbf{r}_i} |\mathbf{r}_i\rangle \langle \mathbf{r}_i| \quad (5)$$

with the broadening constant  $\eta = 1.25t \approx 27.25 \text{ meV}$ , representing metallic contact regions. The sum is over all sites which are coupled to the same contact  $k$ .

The total current at the  $i$ th contact is calculated by the Landauer-Büttiker formula in its linear response approximation

$$I_i = \frac{2e}{h} \sum_j T_{ij} (\mu_j - \mu_i), \quad (6)$$

and the generalized Hall resistance in units of  $h/2e^2$  is given by

$$R_{xy} = \frac{\sum_j (\mathcal{T}_{P_1j} - \mathcal{T}_{P_2j}) T_{jS}}{T_{DS} + \sum_{ij} T_{Di} T_{ij} T_{jS}}, \quad (7)$$

where

$$\mathcal{T}_{ij}^{-1} = \begin{cases} -T_{ij} & i \neq j, \\ \sum_{k \neq i} T_{ik} & i = j. \end{cases} \quad (8)$$

The sums in (7) are over the contacts with unknown chemical potential, whereas the sum in (8) is over all contacts including source and drain.

Finite system size effects, such as standing waves between the boundaries of the system, would distort the focusing spectrum strongly. Therefore, we introduce additional virtual contacts at those boundaries, which are not essential for the focusing experiment, see the gray sites in the inset of Fig. 1. Mathematically these virtual contacts can be considered as additional voltage probes with a chemical potential given by the current conservation constraint. By randomizing the electron phase and momentum,<sup>14,15</sup> such *diffusive walls* mimic an open system and thus, suppress the standing waves. They also greatly reduce spurious focusing peaks arising from reflections at these boundaries.

The local current of electrons, which originate from the source with energy  $\mu$  and which flow from the site  $\mathbf{r}_j$  to the neighboring site  $\mathbf{r}_i$ , is given by<sup>16,17</sup>

$$I_{\mathbf{r}_i \mathbf{r}_j} = \frac{2e}{h} \text{Im} (h_{ji}^* A_{ji}^S), \quad (9)$$

where the  $h_{ij}$  are the matrix elements of the Hamiltonian (2). The spectral function for electrons from the source is defined as

$$A^S = -\frac{2}{\pi} G \text{Im}(\Gamma_S) G^+. \quad (10)$$

The diagonal elements of the spectral function give the local density of states (LDOS), which is accessible to these electrons.

### III. PROPERTIES OF THE SYSTEM

The calculated focusing spectrum, i.e. the generalized Hall resistance  $R_{xy}$  as a function of the magnetic field  $B$  is shown in Fig. 1. In low magnetic fields, equidistant peaks at

$$B_n = \frac{\sqrt{8m\mu}}{e\Delta x} n, \quad n = 1, 2, 3, \dots, \quad (11)$$

are found (vertical dashed lines). As sketched in the inset, electrons injected by the source  $S$  are guided on cyclotron orbits and end in  $P_1$  after  $n - 1$  reflections at the wall in between, when a multiple of the cyclotron diameter equals the distance  $\Delta x$ . These cyclotron orbits can be clearly seen in Fig. 2, which shows the local current and the LDOS of electrons originating from  $S$  with energy  $\mu$ .

When a diffusive wall is introduced also in between  $S$  and  $P_1$ , the higher focusing peaks are strongly suppressed, and the extended plateaus of the quantum Hall

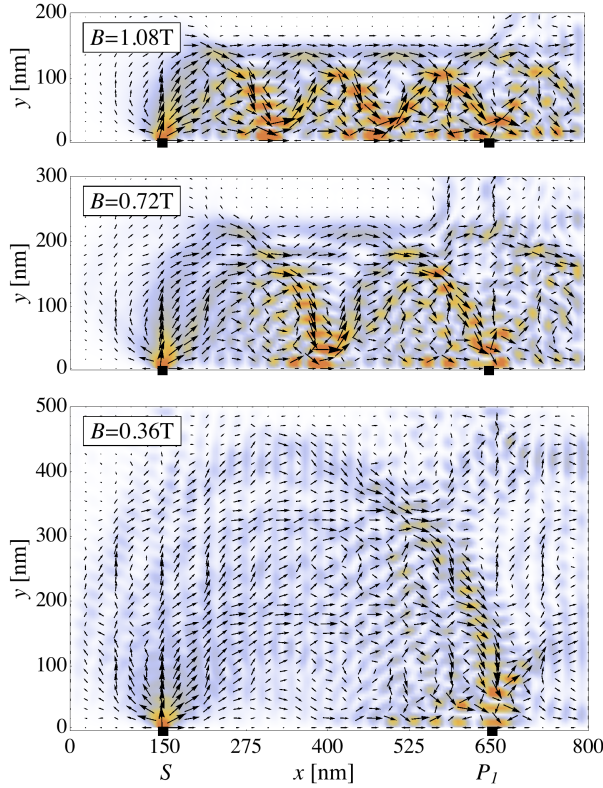


FIG. 2: (Color online) The local current (arrows) and the LDOS (shading) of the electrons originating from  $S$  with energy  $\mu$ . The cyclotron orbits can be clearly seen. Also caustics are evident, which are caused by the interference of the electrons injected with a broad distribution of angles. Note that at  $B = 1.08$  T and  $B = 0.72$  T only the relevant part of the system is shown.

effect appear, see the dashed curve in Fig. 1. The Hall resistance is then given by the inverse number of occupied edge channels, which in turn equals the number of occupied Landau levels. Whenever a Landau level is pushed above the Fermi energy by an increasing magnetic field, an edge channel vanishes and a step in the resistance can be observed.<sup>25</sup>

Because of the coherent injection of electrons with a broad distribution of angles, the local current shows caustics and the focusing spectrum a fine structure. Both effects can be observed experimentally.<sup>2,18</sup> The caustics and the fine structure can be suppressed, if the distribution of the injection angles is narrowed by reducing the distance of the injector to the left diffusive wall, see the focusing spectrum and the local current in Fig. 3.

As expected, Fig. 1 shows that the focusing peaks cannot be observed when the direction of the magnetic field is reversed. The single peak at a low magnetic field is an artifact, which arises when the cyclotron diameter equals the distance between  $S$  and  $P_2$ .

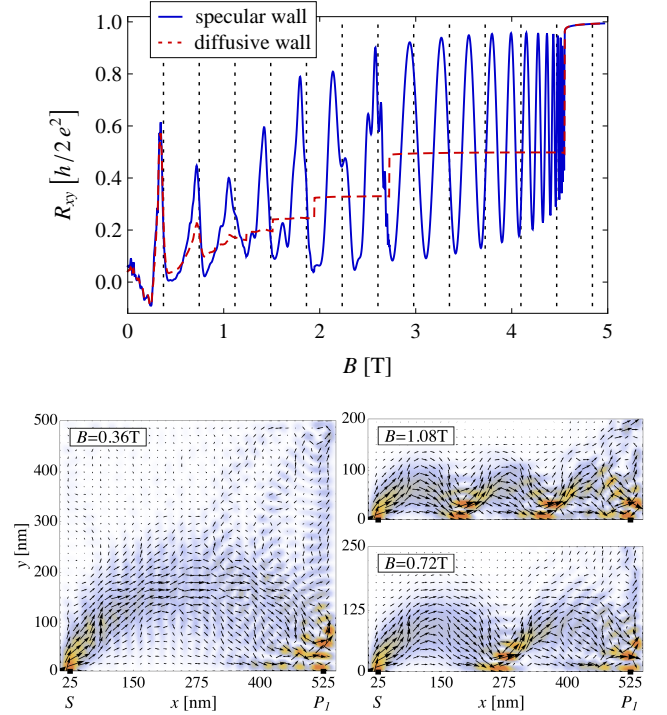


FIG. 3: (Color online) Focusing spectrum and local current, when the distribution of the electron injection angles is narrowed by reducing the distance of the injector to the left absorbing wall. Fine structure and caustics are suppressed.

### A. Anomalous properties of the focusing spectrum

When the strength of the magnetic field is further increased ( $B > 2$  T), we observe an additional set of resistance oscillations, which cannot be explained by classical trajectories. The frequency of these oscillations increases rapidly whenever a Landau level is pushed towards the Fermi energy and a transition between Hall plateaus appears (compare solid and dashed curves in Fig. 1). Moreover, when only two edge channels are occupied ( $2.7$  T  $< B < 4.5$  T), the oscillations become very clear and regular. Finally, the oscillations vanish completely, when only a single edge channel is occupied ( $B > 4.5$  T), and the typical Hall plateau  $R_{xy} = 1$  can be observed.

This suggests that these oscillations are an interference phenomenon between the occupied edge channels as explained in the following. We start by solving the stationary Schrödinger equation with the Hamiltonian (1) and an infinite potential wall along the  $x$ -axis. The Hamiltonian can be rewritten as a harmonic oscillator, which is shifted by  $y_k = \ell^2 k$  with  $\ell^2 = \frac{\hbar}{eB}$  and thus, the edge channels are given by<sup>10</sup>

$$\psi_{k,\nu}(x, y) = c_{k,\nu} e^{ikx} e^{-\frac{1}{2}(y-y_k)^2/\ell^2} H_\nu\left(\frac{y-y_k}{\ell}\right), \quad (12)$$

where  $c_{k,\nu}$  is a normalization constant. Note that the momentum of the plane wave  $e^{ikx}$  parallel to the infinite

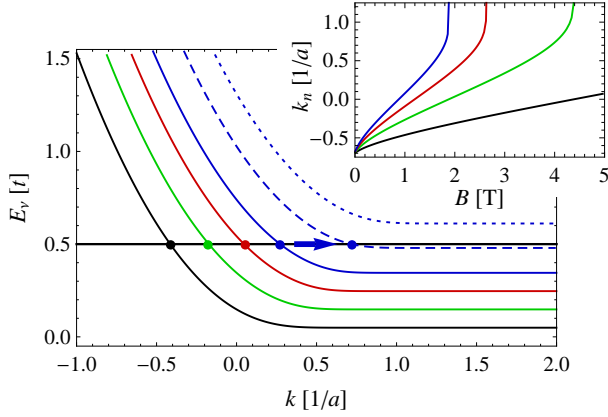


FIG. 4: (Color online) Eigenenergy spectrum (13) of a 2DEG bounded by an infinite potential wall at  $y = 0$ . The solid curves give the first four energy bands at  $B = 1.3$  T while the dashed and the dotted curves give the fourth energy band at  $B = 1.8$  T and  $B = 2.3$  T, respectively. The dots indicate the  $k_n$  at the Fermi energy  $\mu = 0.5t$ . The arrow points out the increase of  $k_4$ , when the corresponding Landau level approaches  $\mu$ . The inset shows the  $k_n$  as a function of  $B$ .

wall determines the apex  $y_k$  of the parabola. The  $H_\nu$  are the Hermite polynomials with index  $\nu$ , which is here in general *non-integer* and which is determined numerically by the hard-wall condition  $\psi_{k,\nu}(x, 0) = 0$ .

The Fig. 4 shows the resulting eigenenergy spectrum

$$E_\nu(k) = \hbar\omega_c (\nu(k) + 1/2), \quad (13)$$

with the cyclotron frequency  $\omega_c = eB/m$ . For sufficiently large  $k$ , the influence of the infinite wall is negligible and we observe the equidistant Landau levels for integer values of  $\nu$ . However, the energy bands are bent upwards, when the apex of the parabola approaches the wall. We numerically calculate the  $k_n$ , which agree with a given Fermi energy and a given magnetic field, see the marked intersection points in Fig. 4. The total number of the  $k_n$  gives the number of occupied edge channels.

We consider only the plane wave contribution in (12), which propagates along the infinite wall, and calculate the superposition of the different  $k_n$  with equal weights. The normalized probability density evaluated at the position of the collector shows remarkable agreement with the NEGF calculation of the focusing spectrum, see the Fig. 5. Thus, the focusing peaks in low magnetic fields, which correspond to classical trajectories, can also be explained by the interference of multiple edge channels.<sup>2,19</sup> Moreover, this explanation of the focusing spectrum is valid for every strength of the magnetic field and allows to understand the anomalous peaks.

In intermediate fields, the current is carried by only a few edge channels and the focusing spectrum shows beatings due to the superposition of plane waves. In particular, when only two edge channels are occupied, only two plane waves are superimposed and a beating appears, whose frequency is determined by the difference

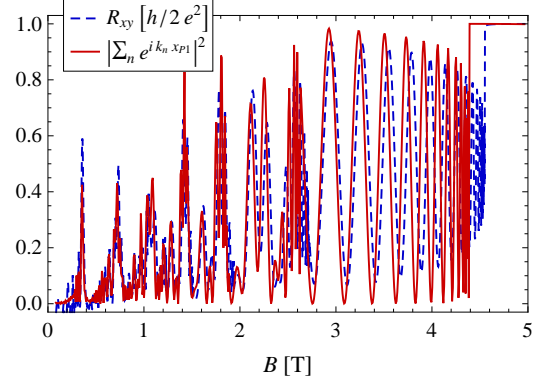


FIG. 5: (Color online) Normalized probability density calculated by a superposition of plane waves with the  $k_n$ , evaluated at the position of the collector  $x_{P1}$  (solid curve). A remarkable agreement with the NEGF calculation of Fig. 1 (dashed curve) can be observed.

of  $k_1$  and  $k_2$ . The frequency of the oscillations increases rapidly, whenever the highest occupied Landau level approaches the Fermi energy, because its intersection point and thus, the corresponding  $k_{\max}$  increases strongly, see the divergences in the inset of Fig. 4. Its difference to the other, much smaller  $k_n$  leads to a high frequency beating. Finally, when only a single edge channel is occupied, the beating and thus, the oscillations in the focusing spectrum vanish. The current then flows along an edge channel parallel to the wall, see the top of Fig. 6. This figure also illustrates that although the focusing peaks in intermediate fields cannot be explained by classical trajectories, the local current resembles to some extent cyclotron motion along the wall.

The clear and distinct oscillations due to the occupation of only two edge channels can also be understood as a new commensurability between the magnetic flux enclosed within the two edge channels and the flux quantum. At the maximum of the oscillations the two plane waves interfere constructively, and thus the difference of their momenta fulfills  $\Delta k = 2\pi/\Delta x$ . If we relate this momentum difference to the distance between the edge channels  $\Delta y_k = \ell^2 \Delta k$ , we obtain

$$\Delta x \Delta y_k B = \frac{h}{e}. \quad (14)$$

Thus, between two successive focusing peaks, the magnetic flux within the area enclosed by the two edge channels increases by one flux quantum. In this way, we can relate the focusing spectrum to the distance of the edge channels and the difference of their momenta. For the experimental observation of such interference effects with a fixed distance between the edge channels, see e.g. Ref. 20 and references therein.

Note that indeed many properties of the system can be understood by a basic quantum calculation. However, this cannot replace the NEGF approach, which allows to



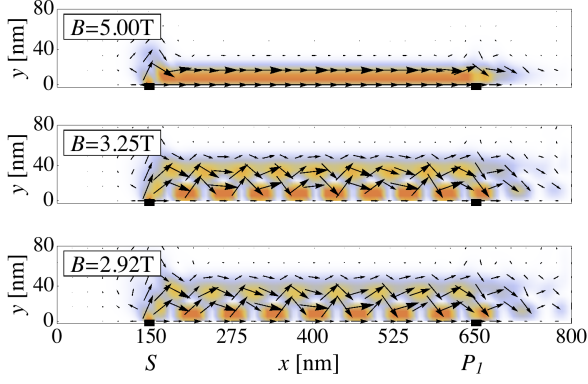


FIG. 6: (Color online) The local current and the LDOS of the electrons originating from  $S$  with energy  $\mu$ . The transport through the interfering edge channels in the lower figures resembles to some extent a cyclotron motion, while at  $B = 5$  T the current is carried through a single edge channel straight along the wall. Only the relevant part of the system is shown.

include contacts in a controlled way and to obtain quantitative results for the Hall resistance. Moreover, the superposition of plane waves with equal weights is justified only by its good agreement with the NEGF calculation.

### B. Effects of decoherence and non-specularity

Dynamic scattering like electron-phonon and electron-electron interaction causes decoherence in the system. We study the effects of weak decoherence on the focusing spectrum by a phenomenological model.<sup>14,15</sup> In the finite-difference approximation of the 2DEG, bonds connecting neighboring sites are selected randomly with probability  $p$  and replaced by virtual contacts. These act as scattering centers, where the electron phase and momentum are randomized completely. The focusing spectrum is averaged over multiple decoherence configurations until convergence is reached.

The averaged focusing spectrum in Fig. 7 shows that with increasing degree of phase and momentum randomization all oscillations are suppressed and the surprisingly robust Hall plateaus appear.<sup>21–23</sup> The classical focusing peaks are even stronger suppressed than the anomalous oscillations, because the latter are located in a much narrower part of the system and thus, are less influenced by the scattering centers. The LDOS and the local current of electrons originating from the source show distinct edge states while the cyclotron orbits are vanishing, because the interference between the edge channels is annihilated by the decoherence. Note that the LDOS is also strongly broadened by the decoherence. As expected, when the degree of decoherence is further increased ( $p \gtrsim 0.05$ ) the quantum Hall plateaus vanish and the classical linear Hall resistance appears.

Our model also allows us to study the effects of partially specular reflections by introducing between  $S$  and

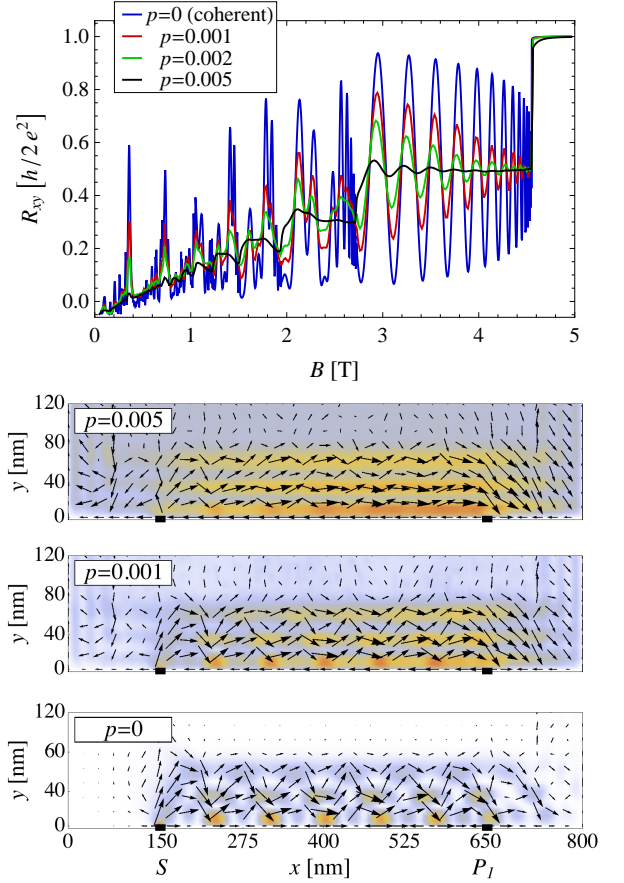


FIG. 7: (Color online) Influence of an increasing degree of decoherence on the focusing spectrum (top) as well as the LDOS and local current (bottom,  $B = 2.13$  T) for electrons originating from the source. The oscillations are gradually suppressed and isolated edge channels remain. Averages are over 75 decoherence configurations.

$P_I$  a diffusive wall with the broadening parameter  $\eta_{dw}$ . In this way, we can tune the scattering from specular ( $\eta_{dw} = 0$ ) to diffusive ( $\eta_{dw} \sim t$ ). The Fig. 8 shows that the oscillations in the focusing spectrum are suppressed gradually with increasing degree of non-specularity and increasing number of reflections at the boundary.

In closing, we discuss requirements to experimentally observe the novel oscillations reported in this paper. In our calculations we have used parameters ( $m = 0.07m_e$ ,  $\mu = 10.9$  meV,  $n_{2D} = 3.3 \cdot 10^{11} \text{ cm}^{-2}$ ), which correspond to a standard 2DEG in a GaAs-AlGaAs heterojunction. We expect that the omission of the spin splitting will not change the results qualitatively. From the fact that the oscillations are seen up to  $p = 0.005$ , see Fig. 7, we infer that a phase coherence length of  $1 \mu\text{m}$  is sufficient for their experimental observation. Likewise, a distance between  $S$  and  $P_I$  of 500 nm is easily achievable with today's nanolithography techniques. All this gives us confidence that the predicted oscillations can indeed be observed experimentally.

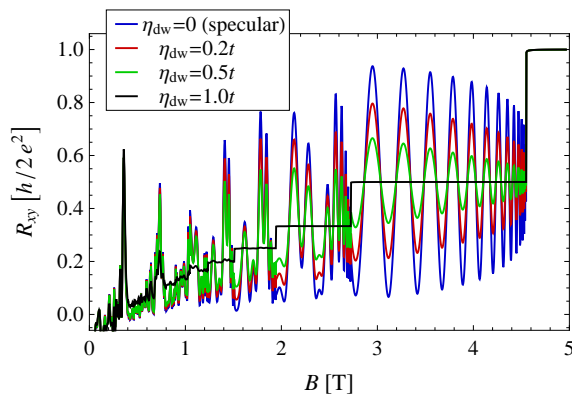


FIG. 8: (Color online) Focusing spectra for an increasingly diffusive boundary. The oscillations in the focusing spectrum are suppressed gradually with increasing degree of non-specularity and increasing number of reflections.

#### IV. SUMMARY

We have studied theoretically the coherent electron focusing in a 2DEG with a boundary. In a weak magnetic field  $B$ , the Hall resistance  $R_{xy}$  shows equidistant peaks, which can be explained by classical trajectories. In a strong field, an extended plateau  $R_{xy} = 1$  reflects the quantum Hall effect. Moreover, in intermediate fields,

superimposed on the lower Hall plateaus we find oscillations, which are neither periodic in  $1/B$  (quantum Hall effect) nor periodic in  $B$  (classical cyclotron motion).

In general, the focusing spectrum can be understood by the interference of the occupied edge channels. In intermediate fields only a few edge channels are occupied and their interference causes beatings. The beatings explain the clear and distinct oscillations in the case of two occupied Landau levels. They constitute a new commensurability between the flux enclosed within the two edge channels and the flux quantum. The frequency of the oscillations increases rapidly when a Landau level approaches the Fermi energy because one of the frequencies contributing to the beating increases strongly. When only a single edge channel is occupied, the beatings and thus, the oscillations in the focusing spectrum vanish abruptly. Weak decoherence suppresses all oscillations in the focusing spectrum and brings out the Hall plateaus.

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- <sup>24</sup> This simplification is justified because in GaAs both, the effective  $g$ -factor and the effective mass are  $\ll 1$  and thus, the spin splitting is typically one order of magnitude smaller than the Landau splitting.
- <sup>25</sup> In the experiment, usually the electron density is constant while the chemical potential is oscillating. However, this would only slightly displace the transitions between the

Hall plateaus and would not qualitatively change our results, see also Ref. 22.